Edge Coloring with Minimum Reload/Changeover Costs

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The cost incurred while traversing a vertex via two consecutive edges of different colors is called *traversal cost*. This cost appeared in the literature under the names of *reload cost* and *changeover cost*. The value of the traversal cost depends on the colors of the incident traversed edges. Although this concept has numerous applications, it has hitherto received little attention. All problems in the literature about the traversal cost concept focus on edge-colored graphs, where the coloring is given as input to the problem. In this work, we take a different approach and focus on proper edge coloring of a given graph such that the total traversal cost is minimized.

The traversal cost of a path is the sum of the costs of each color traversal along the path. The reload cost of a set of paths is the sum of the reload costs of all paths, whereas the changeover cost of a set of paths is the sum of the traversal costs in the set of all traversals. Given a graph, a set of paths, a set of colors, and a traversal cost function for each pair of colors, the minimum reload (resp.changeover) cost edge coloring (MinRCEC resp. MinCCEC) problem aims to find a proper edge coloring of the graph leading to a minimum reload (resp. changeover) cost with respect to the set of paths. Given a graph and a specific vertex called root vertex in the graph, the minimum reload cost path-tree edge coloring (MinRCPTEC) and minimum changeover cost arborescence edge coloring (MinCCAEC) problems aim to find a spanning tree rooted at the root vertex and a proper edge coloring of the tree with minimum total reload and changeover cost, respectively. In this work, we focus on these four problems and have hitherto found the following results:

($\mathcal{P}$ denotes the set of paths, $tc(i,j)$ denotes the traversal cost between color i and color j, X symbolizes the set of colors, and $\Delta(G)$ denotes the maximum degree of the graph G)

Theorem 1 MinCCEC *and* MinRCEC *are inapproximable within any polynomial-time computable function* $f(|\mathcal{P}|)$.

Theorem 2 MinCCEC *and* MinRCEC *are* NP*-Hard in the strong sense even when* $tc(i,j) \in \{1,2\}$ *for every distinct pair* i,j *and* G *is a star.*

Theorem 3 MinCCAEC *and* MinRCPTEC *are* Apx*-Hard in directed graphs even when* $tc(i,j) \in \{1,2\}$ *for every distinct pair* i,j.

Theorem 4 MinCCEC *and* MinRCEC *are solvable in polynomial time when* G *is a tree and* $|X|^{\Delta(G)}$ *is polynomial in the input size.*

Theorem 5 MinCCEC *and* MinRCEC *are solvable in polynomial time when* G *is a tree, and a particular vertex* r *is an endpoint of every path* $P \in \mathcal{P}$.

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