On the Complexity of Constructing Minimum Reload Cost Path-Trees

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Abstract. The reload cost concept refers to the cost that occurs at a vertex along a path on an edge-colored graph when it traverses an internal vertex between two edges of different colors. The reload cost depends only on the colors of the traversed edges. Previous work on reload costs focuses on the problem of finding a spanning tree that minimizes the total reload cost from a source vertex to all other vertices in a directed graph and proves that this problem is not approximable within any polynomial time computable function of the input size. In this paper we study the complexity and approximability properties of numerous special cases of this problem. We consider both directed and undirected graphs, bounded and unbounded number of colors, bounded and unbounded degree graphs, and bounded and unbounded inter-color reload costs. This problem occurs in various applications such as transportation networks, energy distribution networks, and telecommunications.

Keywords:
Reload cost model, approximation algorithms, network design, network optimization.

1 Introduction

1.1 Background

Various network optimization problems can be modeled using edge-colored graphs. In this work we focus on a network design problem under the *reload cost* model, associated with the cost incurred while traversing through a vertex via two consecutive edges of different colors. The reload cost associated with the traversal of this vertex depends only on the colors of the incident traversed edges. Although reload costs have important applications in many areas such as transportation networks, energy distribution networks and telecommunications, they have received little attention in the literature.

Each carrier in an intermodal cargo transportation network can be represented by a color. The cost of transferring cargo from one carrier to another can be modeled by reload costs. In energy distribution networks, transfer of energy from one type of carrier to another has a reload cost. In telecommunications, reload costs arise in many different settings. In heterogeneous networks, routing may necessitate switching from one technology (such as a cellular network) to another (such as a wireless local area network) and this switching cost can be modeled by using the reload cost concept. Even within the same technology, switching between networks of different service providers have different costs corresponding to reload costs [2, 9]. Recently, *dynamic spectrum access (DSA)* networks have been studied in the wireless networking literature [1,8]. Assigned frequency bands in an ad hoc DSA network can be significantly far away from one another. Hence, unlike other wireless networks, switching from one frequency band to another in a DSA network can have a significant cost in terms of delay and power consumption. This frequency switching cost can be modeled using reload costs.

1.2 Related Work

The reload cost concept is introduced in [9] where the focus is on the problem of finding a spanning tree in an edge colored undirected graph having minimum diameter with respect to reload costs. It is proven that, in its most general case, the problem is hard to approximate within any polynomial-time computable function $f(n)$, and is hard to approximate within any constant factor less than 3 even when the instance is restricted to graphs of maximum degree 5. A polynomial time algorithm for graphs with maximum degree at most 3 is also provided. In [5] integer programming formulation is used to minimize the sum of the reload costs of all paths between pairs of vertices in a spanning tree.

The work [2] presents numerous path, tour, and flow problems concerning reload costs. In particular, it focuses on the *Minimum Reload Cost Path-Tree* (MinRcpt) problem, which is to find a spanning tree that minimizes the total reload cost from a source vertex to all other vertices. It is shown that, in a directed graph, MinRcpt is inapproximable within any polynomial-time computable function. Given such an inapproximability result, a natural research direction is to investigate the hardness of the problem in specific cases. Such an approach was taken in [3, 4, 9], although the focus was on other problems.

In [3] the authors study the minimum diameter reload cost spanning tree problem when restricted to graphs with maximum degree 4. In particular, it is proved that if reload costs are unrestricted, the problem cannot be approximated within any constant $c < 2$, and it cannot be approximated within any constant $c < 5/3$ if reload costs satisfy the triangle inequality. [6] studies problems that find a path/trail/walk by minimizing the total reload cost. It focuses on cases where reload costs are symmetric/asymmetric and do/do not satisfy the triangle inequality. [4] focuses on the minimum reload cost cycle cover problem, which aims to span the vertices of an edge-colored graph by a set of vertex-disjoint cycles.

1.3 Our Contribution

In this work we investigate some special cases of MinRcpt depending on parameters such as directed graphs (*digraphs*) and undirected graphs, bounded and unbounded vertex degrees, bounded and unbounded number of colors, and bounded and unbounded reload cost values. We assume without loss of generality that the minimum non-zero reload cost value is 1, and we show the following results:

- There is a constant $\rho > 0$, such that MinRcpt is hard to approximate within $\rho \cdot \log |V|$ in digraphs even when there are only two colors and all reload costs are either 0 or 1.
- There is a constant $\rho > 0$, such that it is hard to approximate MinRcpt within $\rho \cdot \log |V|$ in undirected graphs even when all reload costs are either 0 or 1.
- MinRcpt is in Apx-Hard in both directed and undirected graphs, even when there are only two colors, all reload costs are either 0 or 1, and the maximum degree of the graph is bounded by any constant $B \geq 5$.

We refer to the MinRcpt problem in directed graphs by MinDirectedRcpt. We summarize our results in Table 1. In Section 2 we present our inapproximability results. Finally in Section 3 we discuss further research directions.

Directed Graphs	Bounded Degree	Apx-Hard for 2 colors
	Unbounded Degree	log −Apx-Hard for 2 colors
Undirected Graphs	Bounded Degree	Apx-Hard for 2 colors
	Unbounded Degree	log −Apx-Hard

Table 1. Summary of results

2 Results

We first provide hardness results for digraphs. Our results hold even for the simplest possible reload cost matrix, i.e. a uniform metric with two colors. For general digraphs we show a lower bound of $\Omega(\log(|V|))$, and for digraphs with bounded degree we show that the problem is in APX-Hard.

We consider edge colored graphs $G = (V(G), E(G))$, where the edges are colored with colors from a finite set C of colors. $\Delta(G)$ denotes the maximum degree of G and RC_{max} denotes the maximum reload cost value. Proofs can be found in our technical report [7].

Theorem 1. *[7] There is a constant $\rho > 0$, such that* MinDirectedRcpt *is $\rho \cdot \log |V|$-inapproximable even when $|C| = 2$ and $RC_{max} = 1$.*

Theorem 2. *[7] For any integer constant $B \geq 5$,* MinDirectedRcpt *is in* Apx*-Hard even when $\Delta(G) = B$, $|C| = 2$ and $RC_{max} = 1$.*

The following theorems state the lower bounds for undirected graphs with bounded degree.

Theorem 3. *[7]* MinRcpt *is in* Apx*-Hard even when $\Delta(G) \leq 5$, $|C| = 2$ and $RC_{max} = 1$.*

Theorem 4. *[7] There is a constant $\rho > 0$, such that* MinRcpt *is $\rho \cdot \log(|V|)$-inapproximable even when $RC_{max} = 1$.*

3 Conclusion

We have studied special cases of the minimum reload cost path-tree problem in directed and undirected graphs. We developed hardness results on cases such as bounded and unbounded vertex degrees, bounded and unbounded number of colors and bounded reload cost values. Our results indicate that MinRcpt is inherently difficult to approximate even in special cases.

Two main research directions are subject of work in progress: a) to develop approximation algorithms for the special cases considered in this work, b) to get hardness results for other cases not considered in this work. In particular, it will be interesting to find approximation algorithms as well as stronger inapproximability results for directed graphs with an unbounded number of colors.

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