# An Interference Aware Throughput Maximizing Scheduler for Centralized Cognitive Radio Networks 

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#### Abstract

In this paper, we propose an interference aware throughput maximizing scheduler for cognitive radio networks (CRNs) as part of a MAC layer resource allocation framework. In the considered CRN scenario, the cognitive users with multiple antennas are coordinated by a centralized cognitive base station. We evaluate the performance of our proposed scheme using analysis of variation (ANOVA) technique. We also show experimental results for the total throughput for varying number of cognitive users and frequencies. ${ }^{1}$

Index Terms-Resource allocation, scheduling, MAC, dynamic spectrum access, cognitive radio networks.


## I. INTRODUCTION

Recent studies reveal that the current fixed spectrum assignment policy yields inefficient spectrum usage. To overcome this problem, dynamic spectrum access (DSA) concept has been introduced in the literature. DSA concept relies on the idea of enabling unlicensed users to opportunistically utilize the licensed portions of the spectrum that are spatio-temporally unoccupied by the licensed users. Cognitive radio (CR) technology, initially proposed by [1], is the key enabler of DSA concept. CR is an intelligent device that has the ability to sense and analyze the information about its radio environment.

Cognitive radio networks (CRNs) consist of primary users and secondary users. Primary user (PU) is the licensed owner of a spectrum band and hence has exclusive rights to access it. Secondary user (SU), on the other hand, is an unlicensed user with cognitive capabilities; therefore, it can access the portions of the spectrum temporarily unoccupied by its PU provided that it vacates them as soon as the PU appears. In the rest of this paper, we use the terms cognitive user and secondary user interchangeably.

In this paper, we propose a scheduling scheme for centralized CRNs. Our scheme maximizes the total throughput of the SUs in the service area of a cognitive base station (CBS) while ensuring that the PUs in the service area are not disturbed, no collisions occur among the SUs, and reliable communication of the SUs with the CBS is maintained.

The rest of this paper is organized as follows: Section II describes the related work, while Section III discusses the problem formulation and our proposed solution. Section IV provides the numerical evaluation and simulation results. Finally, Section V concludes the paper.

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## II. RELATED WORK

The scheduling schemes for CRNs differ from the ones for conventional wireless networks in numerous ways. The availability of the frequency channels as well as the maximum power and data rate of the SUs depend widely on the PU activity. This varying channel availability and opportunistic nature of spectral access makes the scheduling schemes for CRNs quite different from the existing scheduling disciplines.

The opportunistic scheduling schemes proposed by the authors in [2] aim to maximize the throughput utility of the SUs subject to maximum collision constraints with the PUs. They assume that a collision occurs if an SU attempts to access a channel already occupied by a PU. Our approach, on the other hand, considers the disturbance of PUs from an interference perspective rather than a collision perspective by taking into account various other factors such as the channel quality between the SU and PU in terms of fading and distance.

The authors in [3] incorporate interference mitigation techniques to the scheduling schemes already existing in conventional wireless networks. Unlike our work, their approach attempts to reduce interference exposed to the primary system without ensuring that the PUs are not disturbed.

The work in [4] focuses on an iterative approach for joint scheduling, power, and bandwidth allocation for centralized CRNs. Unlike our work they do not consider data rate allocation to the SUs in addition to not considering a time slotted system.

## III. PROBLEM FORMULATION AND PROPOSED SOLUTION



Fig. 1: The considered centralized CRN architecture.

We consider a time-slotted centralized CR system, where the CBS controls and guides the cognitive nodes. Figure 1 illustrates the considered network architecture. The scheduler is at the CBS and determines how many packets and with which frequency each SU will transmit in each time slot.


Fig. 2: Framework for our cognitive scheduling mechanism.

Figure 2 illustrates the mechanism for our cognitive scheduling method, where $S U\left(f^{1}\right)$ and $P U_{1}\left(f^{1}\right)$ illustrates that the $S U$ and the $P U_{1}$ transmits using frequency $f^{1}$. Likewise, $P U_{2}\left(f^{2}\right)$ denotes that $P U_{2}$ communicates using frequency $f^{2}$. Therefore, the ongoing communication mechanisms of $P U_{1}$ and $P U_{2}$ are disrupted if any user transmits data using frequency $f_{1}$ and $f_{2}$, respectively, and the interference power received by the PUs is above the maximum tolerable interference power. In essence, we denote by $P_{I F_{\text {max }}}^{f j}$ the maximum tolerable interference power for $P U_{j}$ and frequency $f$. In other words, $P U_{2}$ is not disturbed by the $S U$ in Figure 2 because they operate using different frequencies, whereas it is possible for $P U_{1}$ to be disturbed by the SU since they operate using the same frequency $f^{1}$. Our objective in this work is to determine the transmission power and consequently the data rate of each SU for each frequency and time slot so that the PUs that are actively communicating in that particular frequency are not disturbed.

Let us denote by $x_{i, t}$ the number of packets in the buffer of $\mathrm{SU} i$ at the beginning of time slot $t$, and by $u_{i, t}$ the number of packets transmitted by user $i$ in time slot $t$. Additionally, let $f_{i, t}$ represent the frequency used by user $i$ in time slot $t$. We denote the location of SU $i$ in time slot $t$ by $L_{i t}$, the location of $\mathrm{PU} j$ in time slot $t$ by $L_{j t}$, and the location of the CBS by $L_{C B S}$. Moreover, we represent the fading coefficient of the channel between $\mathrm{SU} i$ and PU $j$ in time slot $t$ by $h_{i j t}$ and the fading coefficient of the channel between $\mathrm{SU} i$ and the CBS in time slot $t$ by $h_{i 0 t}$. Consequently, the vector of buffer states for a total number of $N$ cognitive nodes is $\overline{x_{t}}=\left[x_{1, t}, x_{2, t}, \ldots, x_{N, t}\right]$, and the vector of transmitted packets is $\overline{u_{t}}=\left[u_{1, t}, u_{2, t}, \ldots, u_{N, t}\right]$. Moreover, the
 vector of PU locations is $\overline{L_{t}^{P U}}=\left[L_{1, t}, L_{2, t}, \ldots, L_{M, t}\right]$, where $M$ is the total number of PUs in the coverage area of the CBS. Furthermore, the matrix of fading coefficients for the channels between the SUs and the PUs is $\mathbf{h}_{\mathbf{t}}^{\mathbf{S U}, \mathbf{P U}}=\left[\mathbf{h}_{\mathbf{i j t}}\right]$, which is an $N \times M$ matrix. Likewise, the vector of fading coefficients for the channels between the SUs and the CBS is $\overline{h_{t}^{S U, C B S}}=\left[h_{10 t}, \ldots, h_{N 0 t}\right]$. In line with the fact that FCC has required the CR devices to have geolocation capability in [5],
we assume that the CBS knows $\overline{L_{t}^{S U}}, \overline{L_{t}^{P U}}, L_{C B S}, \mathbf{h}_{\mathbf{t}}^{\mathbf{S U}, \mathbf{P U}}$, and $\overline{h_{t}^{S U, C B S}}$. Therefore, the scheduler's mapping is $\gamma(t)$ : $\left[\overline{x_{t}}, \overline{L_{t}^{S U}}, \overline{L_{t}^{P U}}, L_{C B S}, \mathbf{h}_{\mathbf{t}}^{\mathbf{S U}, \mathbf{P U}}, \overline{h_{t}^{S U, C B S}}\right] \rightarrow\left[\overline{f_{t}}, \overline{u_{t}}\right]$.

We formulate in (1)-(5) the scheduling problem that maximizes the network throughput, while ensuring that the communication of none of the PUs is disturbed, and reliable communication between the SUs and the CBS is achieved:

$$
\begin{gather*}
\max _{\overline{u_{t}}, \overline{t_{t}}} E\left\{\sum_{i=1}^{N} u_{i, t}\right\}  \tag{1}\\
\text { s.t. } P_{r_{j}}^{f t} \leq P_{I F_{m a x}}^{f j} ; \forall j \epsilon \Phi_{C B S}^{f t}, \forall f \epsilon\{1, . ., F\}  \tag{2}\\
u_{i, t}=B \times \frac{T_{s}}{S} \times \ln \left(1+\frac{P_{r_{C B S}}^{i f t}}{\sigma^{2}}\right) ; \forall i \epsilon\{1, . ., N\}  \tag{3}\\
f_{i, t} \neq f_{i^{\prime}, t} ; \forall i, i^{\prime} \epsilon\{1, . ., N\}, i \neq i^{\prime}  \tag{4}\\
u_{i, t} \leq x_{i, t} \tag{5}
\end{gather*}
$$

where $P_{r_{j}}^{f t}$ denotes the power received by PU $j$ through frequency $f$ in time slot $t$, and $P_{I F_{\max }}^{f j}$ symbolizes the maximum tolerable interference power of PU $j$ for frequency $f$. Furthermore, $\Phi_{C B S}^{f t}$ represents the set of PUs that are actively utilizing frequency $f$ in the coverage area of the CBS in time slot $t$, and $P_{r_{C B S}}^{i f t}$ is the power received by the CBS due to the possible transmission of $\mathrm{SU} i$ using frequency $f$ in time slot $t$. Besides, $B$ is the bandwidth, $T_{s}$ is the time slot length, $S$ is the packet size, and $\sigma^{2}$ is the noise power. In the above formulation from (1) to (5),(1) maximizes the expected value of the total number of packets transmitted by all the SUs, and (2) ensures that the interference power values perceived by the PUs due to the transmissions of the SUs are within the tolerable limits. Constraint (3) guarantees the reliable communication between the $\mathrm{SU} i$ and the CBS by having the scheduler to choose the number of packets transmitted, $u_{i, t}$, equal to the Shannon capacity function for a Gaussian channel [6]. Moreover, (4) guarantees that at most one SU can transmit using a certain time slot and frequency combination, and (5) represents the fact that a user cannot transmit more than the number of packets in its buffer at the beginning of the time slot.
To solve the problem in (1)-(5), we firstly find the maximum allowed transmission power for each $\mathrm{SU} i$ and frequency $f$ in time slot $t$, which we denote here by $P_{x m t}^{i f t}$. We use free space path loss and fading in modeling the channel between the SUs and the PUs, as well as between the SUs and the CBS. Therefore, the following relationship holds between $P_{x m t}^{i f t}$ and $P_{r_{j}}^{f t}$.

$$
\begin{array}{r}
P_{r_{j}}^{f t}=P_{x m t}^{i f t} \times\left|A_{i f t}\right|^{2} \\
\left|A_{i f t}\right|=\max _{j \epsilon \Phi_{C B S}^{f t}}\left(\frac{\lambda_{f}}{4 \pi d_{i j t}} \times\left|h_{i j t}\right|\right) \tag{7}
\end{array}
$$

where $\lambda_{f}$ is the wavelength of frequency $f$, and $d_{i j t}$ equals the distance between SU $i$ and PU $j$ in time slot $t$, and $\Phi_{C B S}^{f t}$ is the set of PUs carrying out their communication using frequency $f$ in time slot $t$ in the coverage area of the CBS. Moreover, $h_{i j}$ denotes the fading coefficient of the channel between SU $i$ and PU $j$. In essence, $\left(\frac{\lambda_{f}}{4 \pi d_{i j t}}\right)^{2}$ refers to the path loss of the channel between SU $i$ and PU $j$ due to the free space path
loss formula, and $\left|h_{i j t}\right|$ refers to the fading coefficient of the channel between $\operatorname{SU} i$ and $\mathrm{PU} j$ in time slot $t$. Hence, we denote by $\left|A_{i f t}\right|$ the maximum channel gain among all the channel gains between the $\mathrm{SU} i$ and all the PUs that are actively using frequency $f$ in time slot $t$.

Let us assume, without loss of generality, that $P_{I F_{m a x}}^{f j}$ is constant for all $j$. Therefore, in the sequel, let us use $P_{I F_{\max }}^{f}$ in lieu of $P_{I F_{\max }}^{f j}$. Additionally, assume for simplicity, and yet without loss of generality, that $S=B \times T_{s}$. Hence, the expressions from (8) to (12) in the following holds:

$$
\begin{gather*}
P_{x m t}^{i f t}=\frac{P_{I F_{\max }}^{f}}{\left|A_{i f t}\right|^{2}}  \tag{8}\\
P_{r_{C B S}}^{i f t}=P_{x m t}^{i f t} \times\left|A_{i 0 t}\right|^{2}  \tag{9}\\
\left|A_{i 0 t}\right|=\frac{\lambda_{f}}{4 \pi d_{i 0 t}} \times\left|h_{i 0 t}\right|  \tag{10}\\
P_{r_{C B S}}^{i f t}=P_{I F_{\max }}^{f} \times\left(\frac{\left|A_{i 0 t}\right|}{\left|A_{i f t}\right|}\right)^{2}  \tag{11}\\
U_{i f t}=\left\lfloor\ln \left(1+P_{I F_{\max }}^{f} \times\left(\frac{\left|A_{i 0 t}\right|}{\left|A_{i f t}\right| \times \sigma}\right)^{2}\right)\right\rfloor \tag{12}
\end{gather*}
$$

where $U_{i f t}$ is the maximum number of packets that can be transmitted by SU $i$ using frequency $f$ in time slot $t$, and $d_{i 0 t}$ is the distance between $\mathrm{SU} i$ and the CBS in time slot $t$. Equations (8) and (9) hold because the maximum possible value for $P_{r_{j}}^{f t}$ is $P_{I F_{\max }}^{f}$ due to (2). The floor operator in (12) is necessary because $U_{i f t}$ can naturally only take integer values.

Assume that the network conditions; i.e., the PU and SU locations, the PU spectrum occupancies, and all the channel fading coefficients, are small enough not to have any impact on the $U_{i f t}$ values for a duration of $T$ time slots in the considered centralized CRN cell. Note that because of the floor operator in (12), the schedule length $T$ does not mandate the PU and SU locations as well as the PU spectrum occupancies to remain constant in that time period, but only requires that the change in their values does not alter $U_{i f t}$. The value of $T$, in general, depends on the characteristics of the spectrum environment. For instance, a slowly varying spectrum environment like the TV broadcast bands utilized by an IEEE 802.22 network allows $T$ to have a fairly large value. Hence, instead of $U_{i f t}$, let us use the notation $U_{i f}$, which denotes the maximum number of packets that can be transmitted by $\mathrm{SU} i$ using frequency $f$ in every time slot for a duration of a total number of $T$ time slots. Then, the solution to the problem formulated in (1)-(5) is the same as the solution to the following binary integer linear program (BILP):

$$
\begin{align*}
& \max \left(\sum_{i=1}^{N} \sum_{f=1}^{F} \sum_{t=1}^{T} \frac{U_{i f} X_{i f t}}{T}\right)  \tag{13}\\
& \text { s.t. } \quad \sum_{f} \sum_{t} X_{i f t} \geq 1 ; \forall i \epsilon\{1, . ., N\}  \tag{14}\\
& X_{i f t}+X_{i^{\prime} f t} \leq 1 ; \forall i, i^{\prime} \epsilon\{1, . ., N\}, i \neq i^{\prime}, \forall f, \forall t  \tag{15}\\
& \quad \sum_{f} X_{i f t} \leq \xi_{i} ; \forall i, \forall t \tag{16}
\end{align*}
$$

where $N$ is the total number of nodes, $F$ is the total number of frequencies, $T$ is the total number of time slots. Besides, $X_{i f t}$ is a binary variable such that $X_{i f t}=1$ if user $i$
transmits with frequency $f$ in time slot $t$ and 0 otherwise, and $\xi_{i}$ is the number of transceivers (antennas) of $\mathrm{SU} i$. In this formulation, (14) ensures that every SU is assigned at least one time slot, while (15) makes certain that at most one SU can transmit in a particular time slot and frequency combination, and consequently preventing collisions among the cognitive nodes. Consider the case that two SUs transmit using a certain frequency and time slot. This implies that two SUs will contribute to the value of $P_{r_{j}}^{f t}$ in (2). Consequently, having more than one cognitive user transmit in the same frequency and time slot may increase the aggregate interference experienced at the PU above the maximum tolerable interference limit, $P_{I F_{\text {max }}}^{f j}$. Thus, in addition to avoiding collisions among the SUs, (15) is also essential to guarantee that the aggregate interference at the PUs is within the tolerable threshold. Moreover, (16) represents the fact that a $\mathrm{SU} i$ cannot transmit at the same time using frequencies more than the number of its transceivers, $\xi_{i}$, because each transceiver can tune to at most one frequency at a time.

Once the scheduler determines the $U_{i f}$ values, in general, each node $i$ transmits $\min \left(x_{i, t}, U_{i f}\right)$ number of packets in time slot $t$. We consider traffic in which all flows are continuously backlogged such that the achieved throughput is entirely related to the scheduling process and channel conditions without any variation due to traffic fluctuation. In other words, in the simulations part of this work, it is always true that $x_{i, t}>U_{i f}$; i.e., the cognitive node $i$ always has sufficient number of packets waiting in its buffer to be transmitted to the CBS. This situation is necessary in order to effectively evaluate the performance of the scheduling process by avoiding the possible impacts of the traffic arrival process.
The optimization problem in (13)-(16) is akin to the optimization problem in [7] except that we additionally take the antenna constraints of the SUs into account here via constraint (16). The work in [7] relies on the interference temperature (IT) model proposed by FCC in [8]. Because the IT model requires the measurement of interference temperature at the PUs and setting an upper interference limit on the entire frequency band, it spurred a lot of debate since its inception and received both positive and negative comments. Most of the negative comments were due to the complexity of its practical implementation at the physical layer. Finally, FCC abandoned the IT concept [9]. In this work, on the other hand, we distance ourselves from the IT debate and rely on a much simpler physical layer model. Instead of measuring the interference temperature at all the measurement points (PUs) and setting an upper limit for each frequency band, the CBS in our model only needs to determine whether the PUs are actively using a particular frequency or not. There is a maximum tolerable interference power for each active PU as opposed to each frequency band in the IT model. This can be accomplished using conventional physical and MAC layer spectrum sensing mechanisms in the CRN literature [10],[11].

## IV. NUMERICAL EVALUATION

We consider a CRN cell with radius of 600 meters, simulate it using Java, and obtain the $U_{i f}$ values for 5000 scheduling periods in each set of simulations. We then solve the optimization problem in (13)-(16) using CPLEX [12]. Each scheduling
period consists of $T=10$ time slots each with $T_{s}=100 \mathrm{~ms}$, and hence making the duration of each scheduling period equal to one second. According to the IEEE 802.22 standard, PUs should be detected within two seconds from their appearance [13]; hence, having the scheduling period equal to one second is sufficient for proper operation. We take the noise variance as $\sigma^{2}=10^{-6}$ and the maximum tolerable interference power of active PUs as $P_{I F_{\max }}^{f j}=10$ milliwatts $\forall f$ and $\forall j$. Moreover, we evaluate our mechanism for additive white gaussian noise (AWGN) channels. That is to say, $h_{i j t}=h_{i 0 t}=1 \forall i, j, t$.

The initial locations of the SUs and the PUs in the cell is determined using uniformly random distribution. They move according to random waypoint mobility model; i.e., each node uniformly randomly selects a target point in the cell and moves towards this point with a constant speed. Upon reaching its target position, each node stays there for a certain amount of time and then again selects another target point, etc. In our simulations, we set the duration of stay between movement periods for each node as 10 seconds. We denote the velocity of the PUs by $V_{p}$ and the velocity of the SUs by $V_{s}$. We analyzed the impact of these two parameters on the throughput performance in the cell.


Fig. 3: PU spectrum occupancy model.

We modeled the spectrum usage behavior of the PUs using the finite state model that we illustrated in Figure 3. Each PU is either in the ON state or OFF state. The ON state encompasses one of the F substates, each corresponding to being active using a frequency among a total of F frequencies. The probability of staying in the ON or OFF states is $p_{S}$. While switching from the OFF state to the ON state, the probability of selecting each frequency is equally likely; therefore, probability of transition from OFF state to any frequency is $\left(1-p_{S}\right) / F$. In a slowly varying spectral environment, $p_{S}$ value is usually low; hence, we selected the $p_{S}$ value as 0.9 in our simulations.

We initially utilized experimental design methods to evaluate the impacts of six parameters using analysis of variation (ANOVA) method [14]. We adopted a " $2^{k}$ factorial" experimental design method, where $k=6$ since we evaluate six

TABLE I: Parameter names and low/high values for the $2^{6}$ factorial design

| Parameter Name | Low (-) value | High (+) value |
| :---: | :---: | :---: |
| N (Number of SUs) | 5 | 30 |
| M (Number of PUs) | 5 | 40 |
| F (Number of frequencies) | 3 | 30 |
| $V_{p}$ (Velocity of PUs) | $1 \mathrm{~m} / \mathrm{s}$ | $25 \mathrm{~m} / \mathrm{s}$ |
| $V_{s}$ (Velocity of SUs) | $1 \mathrm{~m} / \mathrm{s}$ | $25 \mathrm{~m} / \mathrm{s}$ |
| $\xi$ (Number of antennas of SUs) | 1 | 5 |

parameters. In other words, we set both low (-) and high (+) values for the $k=6$ parameters and run experiments for all the $2^{6}=64$ possible parameter settings. After implementing the ANOVA method, we determined the statistically significant and insignificant terms. We then run detailed experiments with the statistically significant terms. That is to say, we initially implement factor screening experiments, and then evaluate the impact of the significant factors. The six parameters we considered together with their low and high values are outlined in Table I. For the velocities of the nodes, we consider the case where all the SUs move with the same speed and all the PUs move with the same speed. This implies that the speed of the SUs $\left(V_{s}\right)$ and the PUs $\left(V_{p}\right)$ can in general be different from each other; however, the speed of a SU is the same as the speed of the other SUs. Likewise, the speed of a PU is the same as the speed of the other PUs. For the low values of the velocities, we take $1 \mathrm{~m} / \mathrm{s}$ as a representative of pedestrian speed, whereas we take $25 \mathrm{~m} / \mathrm{s}$ for the high values representing vehicular speed. Additionally, we consider the case where all the SUs have the same number of antennas; i.e., $\xi_{i}=\xi, \forall i$.

The number of samples that we need to take in the experiments in order to obtain a good estimate of the actual mean depends on the variance of the data. If the variance of the data is little, there is no point in running the experiment with too many samples. Especially considering the fact that we solve BILP problems in CPLEX one after another, it is vital to efficiently use the computational resources. In a data set where the variance is known, the number of samples we need to take to ensure that the sample mean is within $\pm E$ of the actual mean with a $100(1-\alpha) \%$ confidence level is as follows [15]:

$$
\begin{equation*}
n=\left\lceil\left(\frac{z_{\alpha / 2} \sigma_{\text {data }}}{E}\right)^{2}\right\rceil \tag{17}
\end{equation*}
$$

Here, $z_{\alpha / 2}$ denotes the upper $\frac{\alpha}{2} \%$ percentile of the standard normal distribution, $n$ represents the sample size, and $\sigma_{d a t a}$ symbolizes the standard deviation of the data. The ceiling operator is necessary because $n$ has to take integer values. Note here that $2 E$ denotes the width of the confidence interval (CI). In our experiments, we take $\alpha=0.05$ and $E=0.5$; in other words, we can say with $95 \%$ confidence level that we are within $\pm 0.5$ of the actual mean in our experiments. Note here that samples in our case correspond to the number of scheduling periods that we run the simulations for.

In our case, however, we do not know the actual standard deviation of our data. Therefore, we make a statistical estimation of the standard deviation ( $\sigma_{\text {data }}$ ) as we take the samples and use the formula in (17) by plugging in our estimated value for $\sigma_{\text {data }}$. In other words, we employ an iterative method. We firstly

TABLE II: ANOVA results

| Source | Sum Squares | Degrees of Freedom | Mean Square Error | F statistic | P-value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $N$ | 10627082.7 | 1 | 10627082.7 | 662798.03 | 0 |
| $M$ | 115397.5 | 1 | 115397.5 | 7197.2 | 0 |
| $F$ | 75508366.3 | 1 | 75508366.3 | 4709363.63 | 0 |
| $V_{p}$ | 716.6 | 1 | 716.6 | 44.69 | 0 |
| $V_{s}$ | 5363.4 | 1 | 5363.4 | 334.51 | 0 |
| $\xi$ | 5654720.5 | 1 | 5654720.5 | 5654720.5 | 0 |
| $N M$ | 6227.3 | 1 | 6227.3 | 388.39 | 0 |
| $N F$ | 10769739.6 | 1 | 10769739.6 | 671695.37 | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $M F V_{p} V_{s} \xi$ | 1.4 | 1 | 1.4 | 0.09 | 0.7693 |
| $N M F V_{p} V_{s} \xi$ | 4.6 | 1 | 4.6 | 0.29 | 0.5907 |
| Error | 296590.8 | 18498 | 16 | - | - |
| Total | 250858751.9 | 18561 | - | - | - |

take 50 samples because according to the central limit theorem at least 40 samples should be taken in order for the formula in (17) to be valid. Afterwards, we calculate the standard deviation of this data with 50 samples and find the value for $n$ using (17). Finally, we take the sample mean of these $n+50$ samples and conclude that this estimate is our final estimate for the mean of the data. In order to verify the validity of our method, we calculate the standard deviation of these $n+50$ samples and find another value for $n$, which we call $n_{\text {new }}$, by using (17). If $n_{n e w}$ is greater than our actual sample size $n+50$, it implies that there is an undesired feature associated with our data, like the samples not being independent of each other. We do not observe this kind of problem in any of our experiments and hence the validity of our estimation procedure for mean throughput is verified.

We implement multiway (n-way) ANOVA technique using MATLAB because we have different sample sizes for each of the 64 experiments. We present in Table II some portion of the resulting ANOVA table due to space constraints. Using a significance level of $\alpha=0.05$, we conclude that the terms with P -value $>0.05$ are statistically insignificant.

Having eliminated some of the statistically insignificant terms using multi-way ANOVA method, we then fit a linear regression model to the rest of the terms, the total number of which is 37 . For each term $x$ of these 37 terms, the regression analysis yields a regression coefficient $r_{x}$ and a $95 \% \mathrm{CI}\left[r_{x}^{l}, r_{x}^{h}\right]$, where $r_{x}^{l}$ and $r_{x}^{h}$ denote the lower and upper bounds, respectively, for the CI of $r_{x}$. The $R^{2}$ statistic for our first regression model is 0.986 ; i.e., the model explains $98.6 \%$ of the variability in the data. The rest of the statistics are F-statistic: 52.6488, P-value: 0 , and an estimate of error variance of 182.6994 . We do not show the first regression model here due to space constraints. If the CI for a regression coefficient contains 0; i.e., $r_{x}^{l}<0<r_{x}^{h}$, we conclude that this term is statistically insignificant since there is a good chance that this regression coefficient might be equal to 0 . Eliminating these terms, we obtain another regression model with the rest of the terms, the total number of which is 14 . The statistics for our second regression model are $R^{2}=0.9675$, Fstatistic:114.4727, P-value:0, and an error variance estimate of 228.3358 . Note that the new model has a significantly reduced number of terms with only a little decrease in the $R^{2}$ value. We then apply the same method of eliminating the terms with $r_{x}^{l}<0<r_{x}^{h}$ and fit a third regression model for the rest of the
terms, the total number of which is 13 . We observe that none of the regression coefficients in this model contains 0 in its CI. Moreover, 13 terms is a sufficiently simple model. The $R^{2}$ statistic for this third model equals 0.9649 , which is adequate for explaining the variability in the data. Therefore, we conclude that this third model is our final regression model and the terms indicated by this model are the statistically most significant ones. We present our final regression model in Table III. The rest of the statistics for this model are F-statistic:116.8798, Pvalue: 0 , and an error variance estimate of 241.6234 . The term $\beta^{2}$ in the model refers to the constant term.

When we analyze this final regression model in Table III, we observe that the most significant 2-way interactions are $N M, N F, N V_{p}, M F, F V_{p}$, and $F \xi$. Therefore, we analyze the impact of these interactions in more detail. Due to space constraints, we present here the results for only the $N F$ interaction. We run the experiments with various values for the $N$ and $F$ parameters between their $(-)$ and (+) values initially indicated in Table I. This way, we are able to examine in more detail how the average total network throughput is influenced by these parameters. For the rest of the parameters; i.e., $M, V_{p}, V_{s}, \xi$ in the $N F$ interaction case, we take their middle value, which is approximately equal to the average of their (-) and (+) values. We outline the values of all the parameters in the detailed experiments in Table IV.

## TABLE III: Final Regression Model

| Term | Coefficient | Lower limit of 95\% CI | Upper limit of 95\% CI |
| :---: | :---: | :---: | :---: |
| $\beta_{0}$ | 139.1349 | 135.2341 | 143.0357 |
| $N$ | 63.2127 | 59.3119 | 67.1135 |
| $M$ | 16.669 | 12.7682 | 20.5698 |
| $F$ | 17.4247 | 13.5239 | 21.3255 |
| $V_{p}$ | 6.1122 | 2.2114 | 10.013 |
| $N M$ | -14.2436 | -18.144 | -10.3428 |
| $N F$ | -13.1142 | -17.015 | -9.2134 |
| $N V_{p}$ | -4.698 | -8.5988 | -0.7972 |
| $M F$ | 10.9865 | 7.0857 | 14.8873 |
| $F V_{p}$ | 9.9355 | 6.0347 | 13.8363 |
| $F \xi$ | 4.2739 | 0.3731 | 8.1747 |
| $N M F$ | -4.9622 | -8.863 | -1.0614 |
| $N F V_{p}$ | -4.6089 | -8.5097 | -0.7081 |

We present in In Figure 4 the results for the $N F$ interaction; i.e., how the average total network throughput is affected for varying $N$ and $F$. In particular, Figure 4(a) shows a three dimensional version where the average total network throughput


Fig. 4: Average total throughput for varying number of SUs $(\mathrm{N})$ and frequencies (F).

TABLE IV: Parameter Values for Detailed Experiments

| Parameter | Value Range | Middle Value |
| :---: | :---: | :---: |
| $N$ | $\{5,10,15, \cdots, 30\}$ | 15 |
| $M$ | $\{5,10,15, \cdots, 40\}$ | 20 |
| $F$ | $\{3,6,9, \cdots, 30\}$ | 15 |
| $V_{p}$ | $\{1,4,7, \cdots, 25\}$ | 13 |
| $V_{s}$ | $\{1,4,7, \cdots, 25\}$ | 13 |
| $\xi$ | $\{1,2, \cdots, 5\}$ | 3 |

is plotted for six different values of the number of SUs $(N)$ and 10 different values for the number of frequencies $(F)$ making a total of 60 experimental results. Figure $4(b)$, on the other hand, shows the same results but in a two dimensional way and for only $F=3,18$, and 30 . We have plotted this additional two dimensional version for better visualization of the average throughput behavior. We can see in these figures that the average throughput is almost invariant for varying $N$ when $F$ is small. This is because the number of resources $F$ in the system is so little that it does not make much difference to have an increasing number of SUs in the system because almost all of the resources are already occupied by all SUs even when the number of SUs is little. As $F$ increases, increasing the number of SUs increases the average network throughput. This increase continues until some point after which the average network throughput saturates.

## V. CONCLUSION

We propose an interference aware throughput maximizing scheduler for centralized CRNs. Our proposed scheduler guarantees that the ongoing communication of the PUs in the service area of the CBS is not disrupted, no collisions occur among the SUs, and reliable communication of the SUs with the CBS is maintained. We initially employ the ANOVA and regression analysis techniques to identify the statistically significant factors. We also fit a regression model with little number of terms yet still explaining $96.49 \%$ of the total variability in the data. We then run detailed simulations to better analyze the impact of the significant factors.

Due to space constraints, in the detailed simulations we have only presented the impact of the total number of SUs (N) and the total number of frequencies (F). As a future work, we plan to investigate the other five factors which were also identified as statistically significant by ANOVA and regression analysis. Moreover, we also plan to design fair schedulers in line with this framework.

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