Edge Colorings, Decompositions and Embeddings

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An edge coloring of a multigraph is (i) *equalized* if the number of edges colored with any two colors differs by at most one, (ii) *balanced* if for each pair of vertices, among the edges joining the pair, the number of edges of each color differs by at most one from the number of edges of each other color, and (iii) *equitable* if, among the edges incident with each vertex, the number of edges of each color differs by at most one from the number of edges of each other color. Motivated by various scheduling and timetabling problems, de Werra showed that every bipartite graph has a balanced, equitable and equalized *k*-edge-coloring for each positive integer *k*. We apply de Werra's theorem, together with network flows to solve problems on decompositions of graphs and hypergraphs, study conditions under which a partial coloring can be extended to a desired decomposition and settle open problems on embedding partial latin squares and latin cubes (Completing partial latin squares is NP-complete).