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The Libre-Office ILP solver

An company has the opportunity of investing in $n \in \mathbb{N}_+$ different projects, if a project j ($j \in \{1, 2, ..., n\}$) is selected it generates a profit $p_j \in \mathbb{R}_+$ and it has a costs $w_j \in \mathbb{R}_+$. The company has a total budget $c \in \mathbb{R}_+$ and its goal is to determine a subset of projects of maximum total profit that respects its budget.

Questions

- 1. Write a integer linear programming (ILP) model to determine a subset of projects of maximum total profit which can be selected by the company, i.e,. a subset of projects of total cost no larger than the budget. Identify the decisions that must be taken and the corresponding decision variables. Identify and comment the objective function of the problem and the constraints.
- 2. Consider now the instance in which there are five projects (n = 5) with the following profits and costs:

j	1	2	3	4	5
p_j	8	6	14	6	2
w_j	3	3	14 9	5	2

The budget of the company is c = 11.

Write the ILP model for this instance and find an optimal solution and the optimal solution value using the ILP solver of Libre-Office.

- 3. Consider now these additional constraints to be added to the ILP model of the previous instance:
 - at most two out of the five projects can be selected;
 - if project 2 is selected, then project 4 must also be selected;
 - if project 1 is selected, then project 3 cannot be selected.

Find an optimal solution and the optimal solution value using the ILP solver of Libre-Office.

Solution

1. We introduce the following *n* binary variables:

$$x_j = \begin{cases} 1 & \text{if project } j \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \quad \forall j \in \{1, 2, \dots, n\},$$

Using these variables, an ILP model for the problem reads as follows:

$$\max \sum_{j=1}^{n} p_{j} x_{j}$$
(1a)
subject to
$$\sum_{j=1}^{n} w_{j} x_{j} \le c,$$
(1b)

$$x_{j} \in \{0,1\}, \qquad j \in \{1,2,...,n\}.$$
(1c)

The objective function (1a) maximizes the total profit of the selected projects. Constraint (1b) ensures that the budget is respected. Finally constraints (1c) define the variables of the ILP model.

2. The ILP model (1) for the specific instance reads as follows:

max	8 <i>x</i> ₁	+6 <i>x</i> ₂	+14 <i>x</i> ₃	$+6 x_4$	+2 <i>x</i> ₅		
	3 x_1	+ 3 x_2	+9 x_3	+ 5 x_4	+ 2 x_5	≤	11
	<i>x</i> ₁ ,	<i>x</i> ₂ ,	<i>x</i> ₃ ,	<i>x</i> ₄ ,	x_5	E	{0, 1}

We now show how to compute an optimal solution of the ILP model for the previous instance using the ILP solver of Libre-Office. The spreadsheet of the model is presented by the following figure, the yellow cells represent the values of the five variables of the ILP model.

	Α	В	С	D	E	F	G	Н	1	J	К		
1	ILP MODEL												
2													
3		x1	x2	x3	x4	x5							
4													
5		8	6	14	6	2							
6								obj function value					
7		0	0	0	0	0		0					
8													
9		3	3	9	5	2		0	<=	11			
10													

Using the SUMPRODUCT function, we can compute the objective function value of the problem in function of the values given to the variables and the objective-function vector as follows:

	А	В	С	D	E	F	G	Н	1	J	K	L
1						ILP	MODEL					
2												
3		x1	x2	хЗ	x4	x5						
4												
5		8	6	14	6	2						
6								obj function value =SUMPRODUCT (B				
		0	0	0	0	0		=SUMPRODUCT(B	5:F5	,\$B	\$7:\$	6F\$7)
8												
9		3	3	9	5	2		0	<=	11		
10												

Using again the SUMPRODUCT function, we can also compute the left-hand-side of the constraint in function of the values given to the variables and the constraint vector as follows:

	Α	В	С	D	E	F	G	н	l J	K	L
1						ILP	MODEL				
2											
3		x1	x2	x3	x4	x5					
4											
5		8	6	14	6	2					
6								obj function value			
7		0	0	0	0	0		0			
8											
9		3	3	9	5	2		=SUMPRODUCT(B	9:F9, <mark>\$</mark>	B\$7:\$	6F\$7)
10											

The ILP solver of Libre-Office is then called as follows:

Tools Window Help	
Spelling	F7
✓ Automatic Spell Checking	Shift+F7
Thesaurus	Ctrl+F7
Language	+
AutoCorrect Options	
✓ AutoInput	
ImageMap	
Redact	
Auto-Redact	
Goal Seek	
Solver	
Detective	•
Scenarios	
Forms	•
Share Spreadsheet	
Protect Sheet	
Protect Spreadsheet Structure	
Macros	•
Extension Manager	Ctrl+Alt+E
Customize	
Options	Alt+F12

The information of the model are given to the solver in the following manner:

- the cell of the objective function value is put into the "Target cell"
- the sense of the objective function is determined in "Optimize results to" ("Maximum" in this case)
- the cells representing the values of the variables of the problem are put into "By changing cells"
- the information related to the constraints of the model are given to the solver in the part "Limiting conditions". Each constraint is inserted by giving two cells, i.e., the left-hand-side of the constraint and the right-and-side of the constraint. Then the type of the constraint is imposed (in this case "≤").
- finally, since all the variables are binary, the cells of the values of the variables are set as "Binary"

	A	В	С	D	E	F	G				Н		1	J	K
1						ILF	P MODE	L							
2															
3		x1	x2	х3	x4	x5									
4															
5		8	6	14	6		2								
6									obj fu	ncti	on v	value			
7		0	0	0	0	(C					(D		
8															
9		3	3	9	5		2					() <=	11	
10															
11							Solver				×				
12			Target	cell	\$H\$7										
13			Optimiz	ze result to	O Max	imum									
14					⊖ Min	imum									
15					🔿 Valı	le of									
16															
17			By chan	ging cells	\$B\$7:	\$F\$7					-				
18				g Conditio	ons										
19				eference			Ċ	Value							
20			\$H\$				<= ▼	\$J\$9							
21			\$B\$	7:\$F\$7			Binary 🔻			4					
22							<= ▼			-					
23							<= ▼								
21	♦ ▶ ▶ + Sheet1	_	Н	elp			Options	Close		Solve	,	-			

Pressing the button"Solve", the solver computes an optimal solution and the optimal solution value of the ILP model as shown in the following figure:

	А	В	С	D	E	F	G	Н	1	J	К
1						ILP	MODEL				
2											
3		x1	x2	х3	x4	x5					
4											
5		8	6	14	6	2	-				
6								obj function va	alue		
7		1	1	0	1	C)		20		
8											
9		3	3	9	5	2	2		<mark>11</mark> <=	11	
10											
11							Solver	8			
12			Target o	ell	\$H\$7						
13			Optimiz	e result to	O Max	imum					
14					⊖ Min	imum					
15							olving Result				
16					Solving s	uccessful	ly finished.				
17			By chan	ging cells	Result: 2	0					
18				g Conditio			- the second second second				
19				eference	want to r	estore p	ep the result or do you evious values?				
20			\$H\$	_	Resto	re Previo	us Keep Result				
21			\$B\$	7:\$F\$7							
22							<= •				
23							<= ▼				
21				-1-			Options				
K ·	Sheet1		H	elp			Options Close	e Solve			

An optimal solution is:

$$x_1^* = 1$$
, $x_2^* = 1$, $x_3^* = 0$, $x_4^* = 1$, $x_5^* = 0$.

The optimal solution value is z(ILP) = 20

3. The ILP model with the additional constraints reads as follows:

max	8 <i>x</i> ₁	$+6 x_2$	$+ 14 x_3$	$+ 6 x_4$	$+2 x_5$		
	3 <i>x</i> ₁	$+3 x_2$	$+9 x_3$	$+5 x_4$	$+2 x_5$	\leq	11
	x_1	+ <i>x</i> ₂	+ <i>x</i> ₃	+ <i>x</i> ₄	+ <i>x</i> ₅	\leq	2
		x_2		- <i>x</i> ₄		\leq	0
	x_1		+ <i>x</i> ₃			\leq	1
	<i>x</i> ₁ ,	<i>x</i> ₂ ,	<i>x</i> ₃ ,	<i>x</i> ₄ ,	<i>x</i> ₅	E	$\{0, 1\}$

The enreadebast of the model is	presented by the following figure:
The spreadsheet of the model is	

	A	В	С	D	E	F	G	н		J	К
1						ILP	MODEL				
2											
3		x1	x2	x3	x4	x5					
4											
5		8	6	14	6	2					
6								obj function value			
7		0	0	0	0	0		0			
8											
9		3	3	9	5	2		0	<=	11	
10		1	1	1	1	1		0	<=	2	
11		0	1	0	-1	0		0	<=	0	
12		1	0	1	0	0		0	<=	1	

The constraints can be inserted altogether as follows:

	A	В	С	D	E	F		G	Н	I	J	К
1						ILP	MOD	EL				
2												
3		x1	x2	х3	x4	х5						
4												
5		8	6	14	6	2						
6									obj function value			
7		0	C	0	0	0			()		
8												
9		<u> </u>		Solver	E	<u></u>		×	()<=	11	
10									()<=	2	
11	Target cell	\$H\$7							()<=	0	
12	Optimize result to	O Max	imum						()<=	1	
13	-	OMini	imum									
14	_	🔿 Valu	ie of									
15	By changing cells	\$B\$7:\$	5F\$7									
16	Limiting Condition	ns										
17	Cell reference		(Operator	Value							
18	\$H\$9:\$H\$12			<= •	\$J\$9	\$J\$12						
19	\$B\$7:\$F\$7			Binary 🔻								
20	_			<= •								
21												
22				<= •								
23	Help			Options		Close	Solv	e				
24							~					

	A	В	С	D	E	F	G	Н		J	К
1						ILP	MODEL				
2											
3		x1	x2	х3	x4	x5					
4											
5		8	6	14	6	2					
6								obj function value			
7		0	0	1	0	1		16	5		
8											
9		2	2	Solver	E	2		11	L <=	11	
10				50000				2	2<=	2	
11	Target cell	\$H\$7						()<=	0	
12	Optimize result to	O Max	imum					1	L <=	1	
13		Mini	mum	_	_						
14		Solving Result 🛛 😪 🔤									
15	By changing cells	By changing cells									
16	Limiting Conditio	Result: 10	5								
17	Cell reference	Do you w	ant to kee	p the resu	lt or do yo	ou l					
18	\$H\$9:\$H\$12	want to r	estore pre	evious valu	les?						
19	\$B\$7:\$F\$7	Resto	re Previou	IS	Keep Res	ult					
20											
21				<= ▼							
22				<= ▼							
23	Help			Options		Close	Solve				
24											

And this is the new optimal solution and the optimal solution value of the ILP model with the additional constraints:

A new optimal solution is:

$$x_1^* = 0, \ x_2^* = 0, \ x_3^* = 1, \ x_4^* = 0, \ x_5^* = 1$$

The new optimal solution value is z(ILP) = 16.