# Domination Parameters in Graphs

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# Definitions

Roman domination

## Roman dominating function

- Cockayne, Dreyer, Hedetniemi, and Hedetniemi in 2004 defined a *Roman dominating function* (RDF) on a graph $G = (V, E)$ to be a function $f : V \to \{0, 1, 2\}$ satisfying the condition that every vertex $u$ for which $f(u) = 0$ is adjacent to at least one vertex $v$ for which $f(v) = 2$. Roman domination in graphs, *Discrete Math.* **278** (2004).

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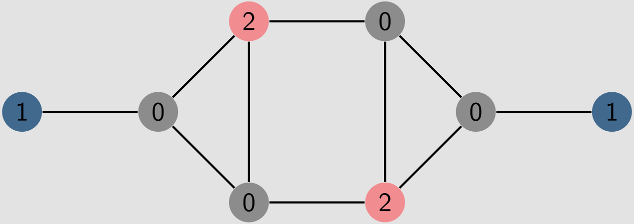

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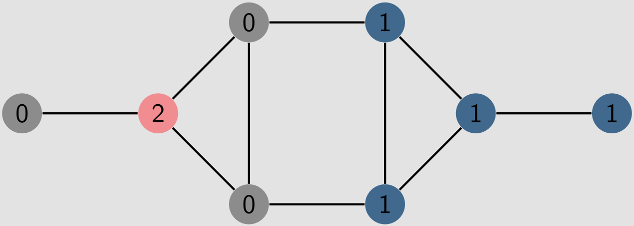

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- For a real–valued function, $f\colon V \to \mathbb{R}$, the *weight* of $f$ is $\mathrm{w}(f) = \bigcup_{v \in V} f(v)$.
- The *Roman domination number*, denoted $\gamma_R(G)$, is the minimum weight of an RDF in $G$; that is, $\gamma_R(G) = \min\{\mathrm{w}(f) : f \text{ is a RDF in } G\}$.

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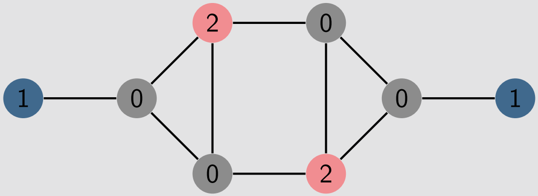


Figure: $\mathrm{w}(f) = 6$

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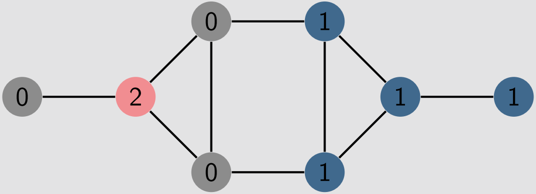


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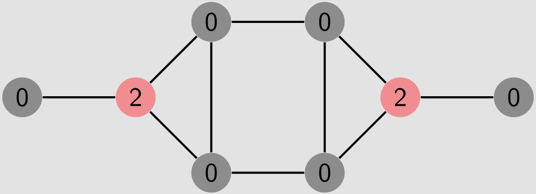


Figure: $\mathrm{w}(f) = 4$, $\gamma_R(G) = 4$

# Historical background

## Roman domination

### Defending Roman Empire

- At the beginning of the 4th century AD, Roman Empire was consolidated and reformed by Emperor Constantine I (306–337).
- He did some remarkable things:
  - reorganized the economy by introducing golden coin solidus (or denarius),
  - allowing religious tolerance (Milan Edict in 312),
  - built new capital (Constantinople),
  - reorganized army.
- Constantine I placed armed forces not only on the borders but throughout all of the Empire.
- Major problem was payment of the army (which was up to 3/4 of the tax revenues), so optimal disposition was of utmost importance.

# Historical background

## Roman domination

### Defending Roman Empire

- Constantine I decided to differentiate communities of the Empire in three categories:
  - those with mobile and stationary troops,
  - those with only stationary troops,
  - those without troops at all.
- The condition was that communities without troops must be in neighborhood of communities with mobile troops, so that in case of attack they can be defended.

# Historical background

Roman domination

## Defending Roman Empire

- Definition of a Roman dominating function was motivated by an article by I. Stewart, Defend the Roman Empire!, Scientific American, 1999.
- Each vertex in graph represents a location in the Roman Empire.
- A location (vertex $v$) is considered *unsecured* if no legions are stationed there (i.e., $f(v) = 0$) and *secured* otherwise (i.e., if $f(v) \in \{1, 2\}$).
- An unsecured location (vertex $v$) can be secured by sending a legion to $v$ from an adjacent location (an adjacent vertex $u$) with two legions.

# Why is it domination?

Roman domination

## Dominating set

- A set $D \subseteq V$ is a *dominating set* of $G$ if each vertex of $V - D$ is adjacent to a vertex of $D$.
- The size of a smallest dominating set of $G$ is the *domination number* of $G$ and is denoted by $\gamma(G)$.

# Properties of Roman domination

## Proposition (Cockayne et al., Roman domination in graphs (2004))

*For any graph $G$,*

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## Roman graphs

- Graphs which have Roman domination number equal to twice their domination number are called *Roman graphs*.

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# Definitions

## Subgraph weakly induced

- Let $G = (V, E)$ be a connected graph and let $S \subseteq V$.
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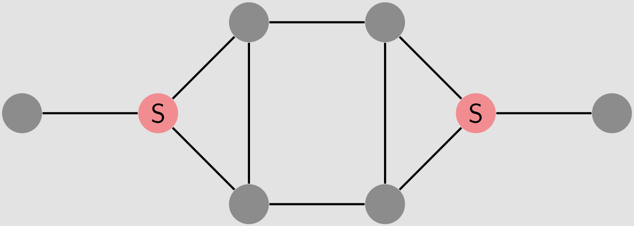


Figure: Graph $G$

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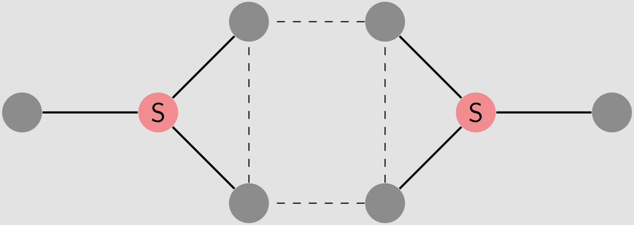


Figure: Graph $\langle S \rangle_w$

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## Weakly connected set

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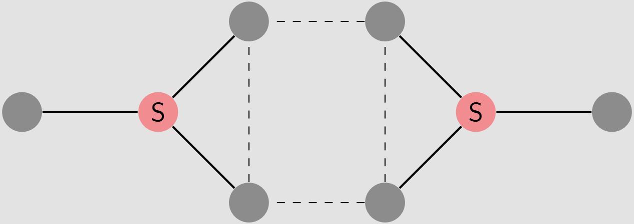


Figure: Graph $\langle S \rangle_w$ is not connected

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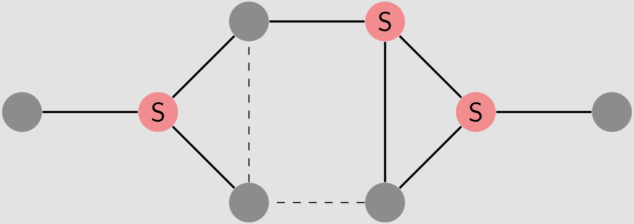


Figure: Graph $\langle S \rangle_w$ is connected

# Definitions

## Weakly connected domination

- A set $D \subseteq V(G)$ is a *dominating set of* $G$ if every vertex $v \in V(G) - D$ is adjacent to a vertex in $D$.
- A set $D \subseteq V(G)$ is a *weakly connected dominating set* (WCDS) of $G$ if $D$ is dominating and $\langle D \rangle_w$ is connected.
- The *weakly connected domination number* of $G$, denoted $\gamma_{wc}(G)$, is the minimum cardinality of a WCDS.

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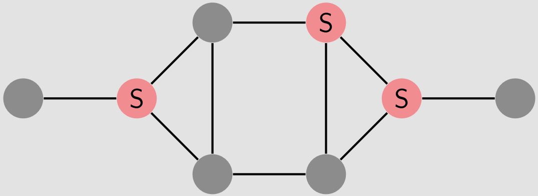

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Weakly connected Roman domination

## Motivation

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*The single biggest problem in communication is the illusion that it has taken place.* George Bernard Shaw

# Definitions

Weakly connected Roman domination

## Motivation

- Two legions at different location (vertices $u$ and $v$) can *contact directly* if there is at most one unsecured location between them and the distance between $u$ and $v$ is at most 2 and they can *contact undirectly* if there is a sequence of vertices $(u = u_1, u_2, \ldots, u_k = v)$ such that $u_i$ and $u_{i+1}$ can contact directly for $i = 1, 2, \ldots, k-1$.

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- The Roman Empire is *communicated* if any two legions at different locations can contact directly or undirectly.

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## Motivation

- A location of a legion is called *uncommunicated* if there exists another location of a legion such that the legions cannot contact directly nor undirectly.

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- If the locations are uncommunicated, they cannot quickly inform the other locations nor ask them for help in case of urgent emergency.
- When all locations of legions are communicated, one can defend the Roman Empire more efficiently: supervise whole Empire and send orders to legions in reasonable time.

# Definitions

Weakly connected Roman domination

## Weakly connected Roman dominating function

- We call the function $f$ a *weakly connected Roman dominating function* in $G$ (WCRDF) if each vertex $u \in V_0$ is adjacent to a vertex $v \in V_2$ and the subgraph $\langle V_1 \cup V_2 \rangle_w$ weakly induced by $V_1 \cup V_2$ is connected in $G$.

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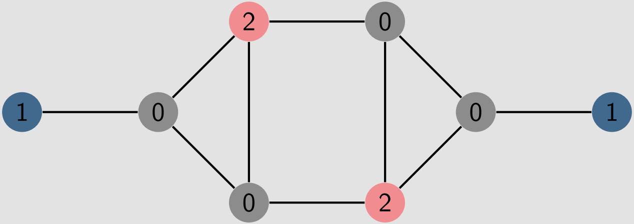

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- We define the *weight* $\mathrm{w}(f)$ of $f$ to be $|V_1| + 2|V_2|$.
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- A WCRDF of weight $\gamma_R^{wc}(G)$ we call a $\gamma_R^{wc}(G)$–function.

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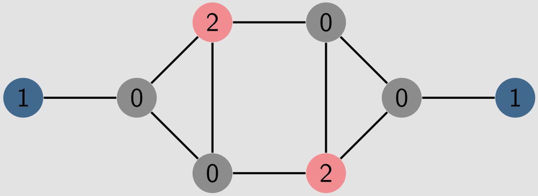


Figure: $w(f) = 6$

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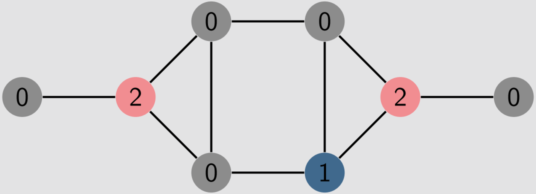


Figure: $\mathrm{w}(f) = 5, \gamma_R^{wc}(G) = 5$

# Bounds

## Proposition

*If $G$ is a connected graph, then*

$$\gamma_{wc}(G) \leq \gamma_R^{wc}(G) \leq 2\gamma_{wc}(G).$$

*The equality in the lover bound holds if and only if $G = K_1$.*

# Bounds

## Proposition

*For any connected graph $G$ of order $n$,*

$$\gamma_R^{wc}(G) \leq n.$$

*The equality $\gamma_R^{wc}(G) = n$ holds if and only if $G \in \{K_1, K_2\}$.*

# Bounds

## Proposition

*If $G = (V, E)$ is a connected graph of order at least 3 and $f = (V_0, V_1, V_2)$ is a $\gamma_R(G)$–function, then*

$$\gamma_R(G) \le \gamma_R^{wc}(G) \le \gamma_R(G) + |V_2| - 1.$$

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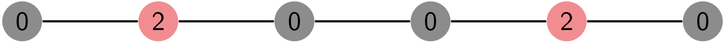

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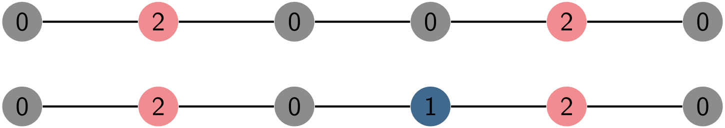

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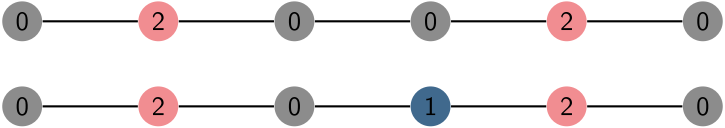


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# Certified dominating set

## Certified dominating set

- A subset $S$ of $V(G)$ is a *certified dominating set* of $G$ if $S$ is a dominating set and every vertex belonging to $S$ has either zero or at least two neighbours in $V(G) - S$.
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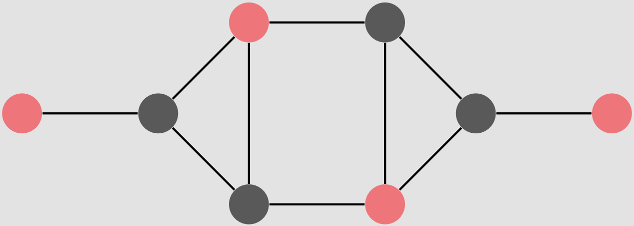

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- The cardinality of a minimum certified dominating set in $G$ is called the *certified domination number* of $G$ and is denoted $\gamma_{cer}(G)$.
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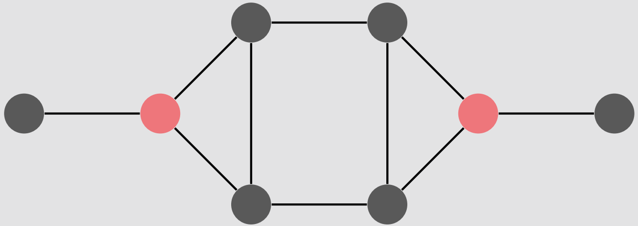

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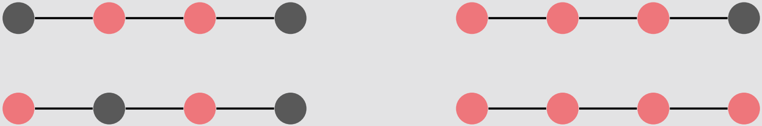
- Every support vertex of a graph $G$ belongs to every certified dominating set of $G$.



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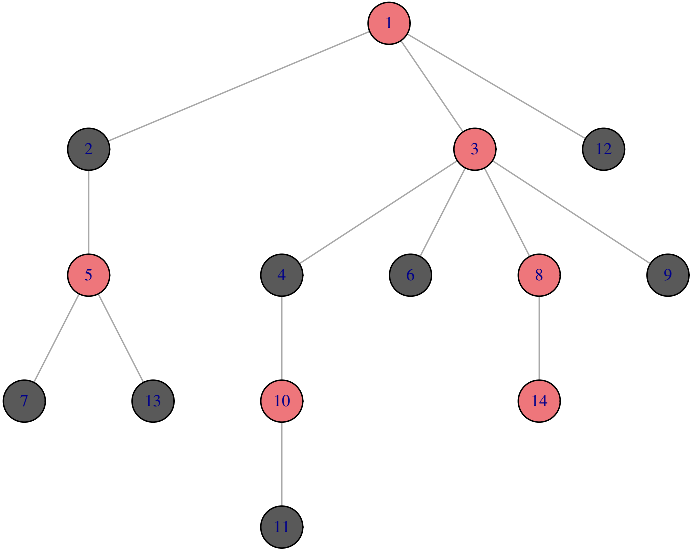

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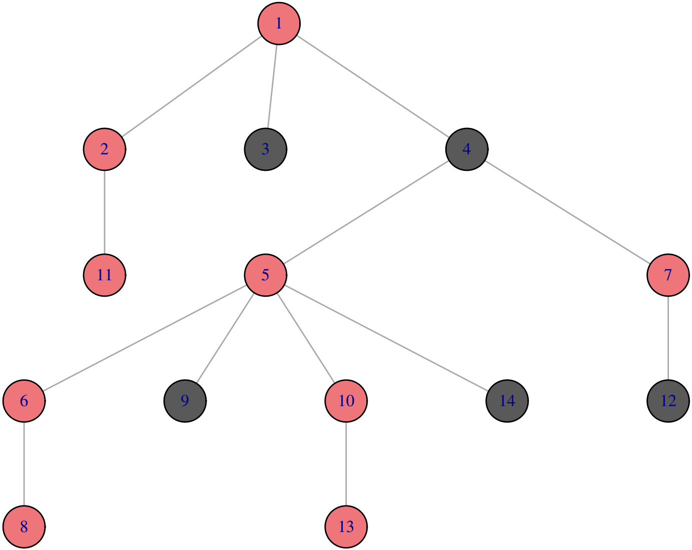

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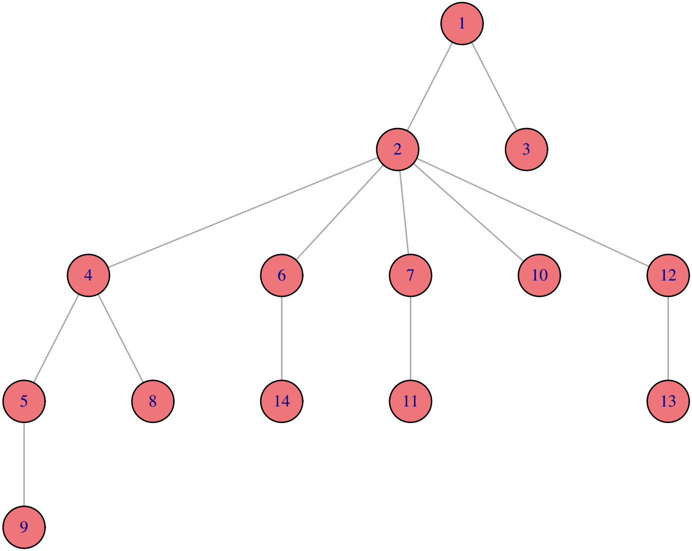

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## Theorem (Dettlaff et al. (2018))

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# Certified domination

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Menti 3

# Secondary Domination

## Motivation

- The secondary domination in graphs was defined by Hedetniemi, Rall et al., Secondary domination in graphs, AKCE Int. J. of Graphs and Comb. (2008).
- The authors of this paper mention a forthcoming paper on the algorithmic complexity of this domination problem, however this paper was never written.
- Since then some papers on this topic have appeared.

# Definitions

## $(1,2)$-domination

- A set $D \subseteq V(G)$ is a $(1,2)$-dominating set if each vertex $v$ of $V - D$ has a neighbour in $D$ as well as another vertex of $D$ at a distance not greater than 2 from $v$.

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- The $(1,2)$-domination in graphs is a special case of $(1,k)$-domination. Here we only deal with the case when $k = 2$, that is, secondary domination.

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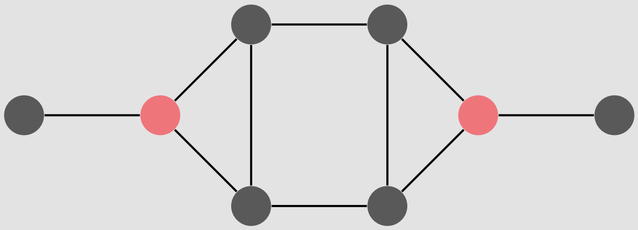

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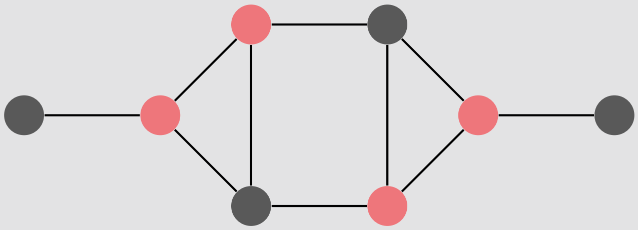

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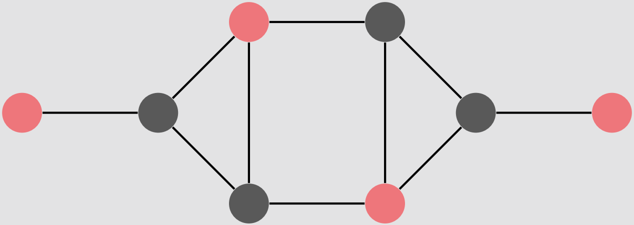

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Menti 4

# Thank you!