Domination Parameters in Graphs

Tutorials. Prove or disprove.

Bounds

1. For a graph G, $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$.
2. For a graph G, $\gamma(G) \leq \gamma_R^{wc}(G) \leq 2\gamma(G)$.
3. For a graph G, $\gamma(G) \leq \gamma_{cer}(G) \leq 2\gamma(G)$.
4. For a graph G, $\gamma(G) \leq \gamma_{1,2}(G) \leq 2\gamma(G) - 1$.
5. For a graph G, $\gamma_{1,2}(G) \leq \gamma_R^{wc}(G) \leq 2\gamma_{1,2}(G)$.
6. For a graph G, $\gamma_{cer}(G) \leq \gamma_R^{wc}(G) \leq 2\gamma_{cer}(G)$.

Equalities

1. If $\gamma_{1,2}(G) = 2\gamma(G) - 1$, then G is Roman.
2. If G is Roman, then $\gamma_{1,2}(G) = 2\gamma(G) - 1$.
3. If G is Roman, then $\gamma_{cer}(G) = \gamma(G)$.
4. If $\gamma_{cer}(G) = \gamma(G)$, then G is Roman.
5. If G is without leaves, then $\gamma_{cer}(G) = \gamma(G)$.
6. If $\gamma_{cer}(G) = \gamma(G)$, then G is without leaves.
7. If G is without leaves and without triangles, then $\gamma_{1,2}(G) = \gamma(G)$.
8. If $\gamma_{1,2}(G) = \gamma(G)$, then G is without leaves and without triangles.

Trees I

1. If T is a tree with $n(T) \geq 3$, then $\gamma_{1,2}(T) \leq \gamma_{cer}(G)$.
2. If T is a tree with $n(T) \geq 3$, then $\gamma_{1,2}(T) \leq \gamma_R^{wc}(G)$.
3. If T is a tree with $n(T) \geq 3$, then $\gamma_R(T) \leq 2\gamma_{cer}(G)$.

Trees II

- Let $n_1(T)$ be the number of leaves in T.
- It is proven that $\gamma(T) \geq \frac{n(T)-n_1(T)+2}{3}$.
- Find similar bounds for other domination numbers.