

The Art of Coloring Graphs: Models, Methods and Applications

Hanna Furmańczyk

Institute of Informatics University of Gdańsk, Poland

Abstract

Graph coloring is one of the fundamental topics in graph theory, with a rich history and numerous applications in scheduling, frequency assignment, resource allocation, and network design. Starting with the classical map-coloring problem, this talk explores how changing the notion of a “good” coloring leads to a variety of different mathematical models and algorithmic challenges.

We first recall the basic concepts and properties of proper vertex coloring and then move beyond the classical setting to several non-standard coloring models, including equitable coloring, packing coloring, majority c -coloring, and b -coloring. These models illustrate that the objective of a coloring problem need not always be the same: while in many settings we seek to minimize the number of colors, in others we aim to maximize it, or impose additional structural or local requirements on the color classes.

A particular focus of the talk is the algorithmic and computational complexity of these problems. We discuss not only classical NP-hardness results, but also what can still be achieved when a problem is computationally difficult. Parameterized complexity, structural graph properties, bounds, and specialized algorithms provide ways of exploiting additional structure and obtaining meaningful results beyond a simple classification as “hard.”

Through selected examples and recent results, the talk aims to present graph coloring not as a single problem, but as a broad collection of models in which the choice of constraints, objectives, and parameters fundamentally changes the questions we ask—and the methods we can use to answer them.

References

- [1] C. Bujtas, M. Dettlaff, H. Furmańczyk, A. Laskowska, Majority C-coloring of graphs, arXiv preprint arXiv:2604.20752 (2026).
- [2] M. Dettlaff, H. Furmańczyk, I. Peterin, A. Roux, R. Ziemann, A new approach to b-coloring of regular graphs, *Applied Mathematics and Computation* 481(2024).

- [3] H. Furmańczyk, D. Gözüpek, S. Özkan, Packing coloring of graphs with long paths, <https://arxiv.org/abs/2511.12761>.
- [4] H. Furmańczyk, V. Mkrtchyan, Graph theoretic and algorithmic aspect of the equitable coloring problem in block graphs, *Discrete Mathematics & Theoretical Computer Science*, 23(2) (2022).
- [5] M. Kubale ed., *Graph Colorings*, Contemporary Mathematics 352, AMS, Ann Arbor (2004).
- [6] N. Robertson, D. Sanders, P. Seymour, R. Thomas, The four-colour theorem, *Journal of Combinatorial Theory, Series B* 70 (1997), 2–44.