

# Modifying Graph Parameters Via Graph Operations

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A typical graph modification problem aims to modify a graph  $G$ , via a small number of operations from a specified set  $S$ , into some other graph  $H$  that has a certain desired property, which usually describes a certain graph class  $\mathcal{G}$  to which  $H$  must belong. In this way a variety of classical graph-theoretic problems is captured. For instance, if only  $k$  vertex deletions are allowed and  $H$  must be an independent set or a clique, we obtain the Independent Set or Clique problem, respectively.

Now, instead of fixing a particular graph class  $\mathcal{G}$ , we fix a certain graph parameter  $\pi$ . That is, for a fixed set  $S$  of graph operations, we ask, given a graph  $G$ , integers  $k$  and  $d$ , whether  $G$  can be transformed into a graph  $G'$  by using at most  $k$  operations from  $S$ , such that  $\pi(G') \leq \pi(G) - d$ . The integer  $d$  is called the *threshold*. Such problems are called *blocker problems*, as the set of vertices or edges involved “block” some desirable graph property, such as being colourable with only a few colours. Identifying the part of the graph responsible for a significant decrease of the parameter under consideration gives crucial information on the graph.

In this lecture, we present result regarding blocker problems for the chromatic number, the matching number as well as for different types of domination parameters.