Dominating Sequences in Graphs

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Let $G = (V, E)$ be a simple graph. A subset D of V is a dominating set of G if each vertex not in D is adjacent to a vertex in D. The *domination number* $\gamma(G)$ of G is the size of a smallest dominating set of G. A sequence of vertices in a graph G is called a *legal dominating sequence* if every vertex in the sequence dominates at least one vertex not dominated by those vertices that precede it, and at the end all vertices of G are dominated. While the length of a shortest such sequence of a graph G corresponds to $\gamma(G)$, the size of a legal dominating sequence of maximum length is called the *Grundy domination number* of G denoted by $\gamma_{gr}(G)$.

If a graph $G=(V, E)$ has no isolated vertices, a subset D of V is called a *total dominating set* of G if every vertex of V is adjacent to at least one member of D. The size of a smallest total dominating set of G is called the *total domination number* $\gamma_t(G)$ of G. A sequence of vertices in a graph G without isolated vertices is called a *total dominating sequence* if every vertex v in the sequence has a neighbor which is adjacent to no vertex preceding v in the sequence, and at the end every vertex of G has at least one neighbor in the sequence. Minimum length of such a sequence in a graph G corresponds to $\gamma_t(G)$, while the maximum length of such a sequence is called the *Grundy total domination number* $\gamma_{gr}^t(G)$ of G. In these lecture series, we will review results on Grundy domination number and Grundy total domination number as well as the studies on graphs with equal domination and Grundy domination numbers, that is, graphs G with $\gamma(G)= \gamma_{gr}(G)=k$, referred to as *k-uniform graphs,* and graphs with equal total domination and Grundy total domination numbers, that is, graphs G with $\gamma_t(G)= \gamma_{gr}^t(G)=k$, referred to as *total k-uniform graphs*.