Domination Parameters in Graphs

Definitions

Domination

- A set $D \subseteq V$ is a *dominating set* of $G = (V, E)$ if each vertex of $V - D$ is adjacent to a vertex of D.
- The size of a smallest dominating set of G is the *domination number* of G and is denoted by $\gamma(G)$.

Roman Domination

- A *Roman dominating function* (RDF) on a graph G is a function $f: V \to \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$.
- The *Roman domination number*, denoted $\gamma_R(G)$, is the minimum weight of an RDF in G; that is, $\gamma_R(G) = \min\{\bigcup_{v \in V} f(v) : f \text{ is a RDF in } G\}$.

Weakly Connected Domination

- The *subgraph weakly induced* by D is the graph $\langle D \rangle_w = (N[D], E_w)$, where E_w consists of all edges in E having at least one vertex in D.
- A set D is a weakly connected set if $\langle D \rangle_w$ is connected.
- A set $D \subseteq V$ is a *weakly connected dominating set* (WCDS) of G if D is dominating and $\langle D \rangle_w$ is connected.
- The *weakly connected domination number* of G, denoted $\gamma_{wc}(G)$, is the minimum cardinality of a WCDS.

Weakly Connected Roman Domination

- A function $f : V \to \{0, 1, 2\}$ is a *weakly connected Roman dominating function* in G (WCRDF) if each vertex $u \in V_0$ is adjacent to a vertex $v \in V_2$ and the subgraph $\langle V_1 \cup V_2 \rangle_w$ is connected in G.
- The *weakly connected Roman domination number*, denoted $\gamma_R^{wc}(G)$, is the minimum weight of a WCRDF in G.

Certified Domination

- A subset D of V is a *certified dominating set* of G if D is a dominating set and every vertex belonging to D has either zero or at least two neighbours in $V - D$.
- The cardinality of a minimum certified dominating set in G is called the *certified domination number* of G and is denoted $\gamma_{cer}(G)$.

$(1, 2)$-Domination

- A set $D \subseteq V$ is a $(1, 2)$-dominating set if each vertex v of $V - D$ has a neighbour in D as well as another vertex of D is at a distance not greater than 2 from v.
- The $(1, 2)$-domination number, denoted by $\gamma_{1,2}(G)$, is the cardinality of a smallest $(1, 2)$-dominating set of G.