

Structural Parameters and Decompositions for the Maximum Weight Independent Set Problem

by Clément Dallard

The Maximum Weight Independent Set (MWIS) problem is NP-hard on general graphs but becomes polynomial-time solvable for classes that admit suitable decompositions. For example, every chordal graph admits a clique tree over its maximal cliques, enabling a simple dynamic programming (DP) algorithm for MWIS. Generalizing this idea leads to tree decompositions, a type of graph decomposition where vertex subsets (“bags”) are associated with nodes of a tree, allowing DP techniques to be applied to solve MWIS. The efficiency of such algorithms typically depends on the maximum bag size, captured by the notion of treewidth. Yet, treewidth alone does not always reflect the true algorithmic barrier. By modifying the definition of treewidth to measure the independence number of a bag instead of its size, one obtains the tree-independence number, a graph parameter that strictly generalizes treewidth. Importantly, MWIS can be solved in polynomial time on every graph class of bounded tree-independence number.

During the lecture, we will progress from chordal graphs and clique trees to tree decompositions and treewidth, and then introduce the tree-independence number together with its algorithmic and structural implications.