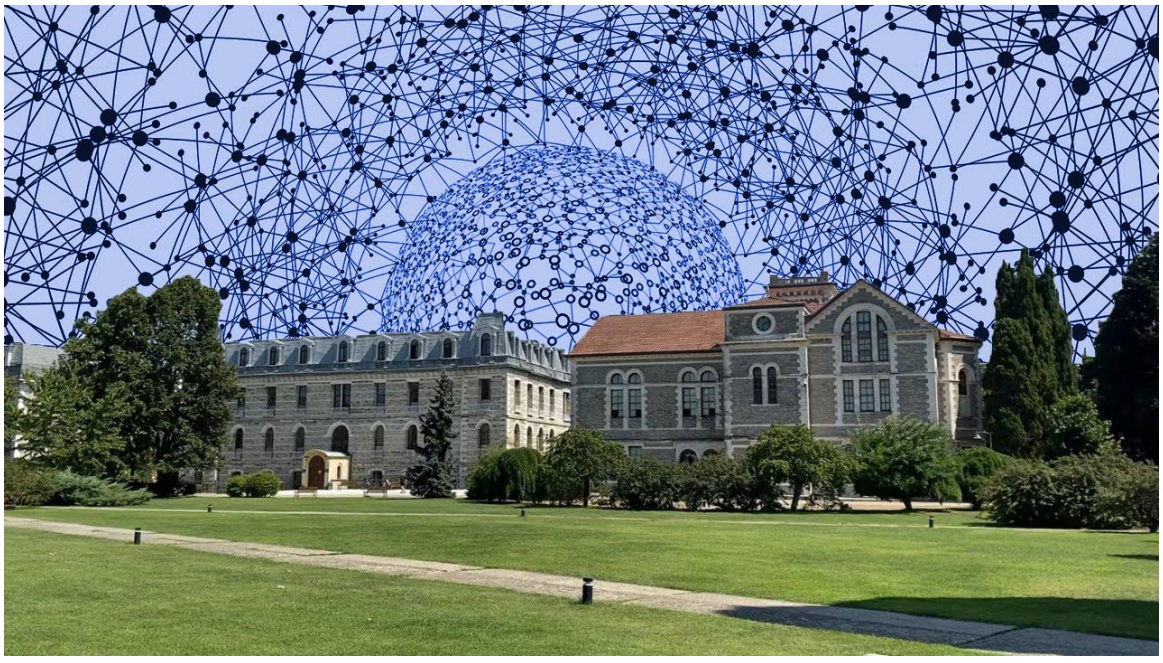
Workshop on Graph Theory and Its Applications (WGT-XIII)

ABSTRACT BOOKLET

16-17 October 2026

Boğaziçi University, Istanbul, Turkey



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Invited lectures: Hanna Furmańczyk (16 October) and Miklós Krész (17 October)

This workshop is conducted as part of the 20th anniversary of the Istanbul Center for Mathematical Sciences (IMBM).

The Art of Coloring Graphs: Models, Methods and Applications

Hanna Furmańczyk

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Invited Lecture — Friday, 16 October 2026

Graph coloring is one of the fundamental topics in graph theory, with a rich history and numerous applications in scheduling, frequency assignment, resource allocation, and network design. Starting with the classical map-coloring problem, this talk explores how changing the notion of a "good" coloring leads to a variety of different mathematical models and algorithmic challenges.

We first recall the basic concepts and properties of proper vertex coloring and then move beyond the classical setting to several non-standard coloring models, including equitable coloring, packing coloring, majority c-coloring, and b-coloring. These models illustrate that the objective of a coloring problem need not always be the same: while in many settings we seek to minimize the number of colors, in others we aim to maximize it, or impose additional structural or local requirements on the color classes.

A particular focus of the talk is the algorithmic and computational complexity of these problems. We discuss not only classical NP-hardness results, but also what can still be achieved when a problem is computationally difficult. Parameterized complexity, structural graph properties, bounds, and specialized algorithms provide ways of exploiting additional structure and obtaining meaningful results beyond a simple classification as "hard."

Through selected examples and recent results, the talk aims to present graph coloring not as a single problem, but as a broad collection of models in which the choice of constraints, objectives, and parameters fundamentally changes the questions we ask—and the methods we can use to answer them.

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Well Vertex-Edge-Dominated Graphs: Structural Results and Characterizations

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A graph is called well vertex-edge-dominated if all its minimal vertex-edge dominating sets have the same size. In this study, the structural properties of these graphs are examined. First, a general method for constructing well vertex-edge-dominated graphs is presented. The effect of reducing false twin vertices and applying C_4-expansions is also studied. Then, unicyclic graphs are examined. Reduction methods based on removing bridges and certain pendant edges are considered. Using these methods, it is shown that the height of the branches attached to the cycle in the reduced graph is at most two. The types of local structures that can be added to the vertices on the cycle are also identified. These results help us better understand the structural properties of well vertex-edge-dominated unicyclic graphs.

Packing Colorings: Bounds, Vertex Deletion, and Complexity

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A packing coloring of a graph assigns positive integer colors to its vertices so that any two vertices receiving color i are at distance greater than i. The packing chromatic number is the smallest number of colors needed for such a coloring.

In this talk, we first establish an upper bound on the packing chromatic number in terms of the maximum sizes of metric balls.

We next study the packing coloring gap, the largest drop in the packing chromatic number caused by deleting one vertex. We show that this gap equals one for split graphs containing an edge and establish the sharp bound $\mu_\rho(T) \leq \lfloor (n-1)/4 \rfloor$ for trees of order $n \geq 9$.

Finally, we prove that deciding whether a starlike graph admits a packing coloring with at most k colors is NP-complete when k is part of the input. For connected primitive starlike graphs, we give an exact formula for the packing chromatic number and a polynomial-time algorithm constructing an optimal packing coloring.

Packing Coloring Gap of Graphs

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The packing chromatic number of a graph is the minimum number of colors for which the graph admits a packing coloring. This distance-based parameter may change under local structural modifications of the graph. We introduce the packing coloring gap, defined as the maximum decrease in the packing chromatic number resulting from the deletion of a single vertex. We study the packing coloring gap of several graph classes. In addition, we present examples of graphs with packing coloring gap zero, one, and arbitrarily large.

Monophonic visibility in graphs

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We introduce monophonic visibility as a natural variant of mutual visibility. Two vertices $u,v \in S \subseteq V(G)$ are monophonically S-visible if there exists a (u,v)-path whose internal vertices do not belong to S. The set S is a monophonic visibility set if every pair of vertices in S is monophonically S-visible. The maximum cardinality of such a set is denoted by $\mu_m(G)$. In this talk, we investigate the relationship between $\mu_m(G)$ and the mutual-visibility number $\mu(G)$, establish bounds in terms of standard graph invariants, and discuss the computational complexity of determining $\mu_m(G)$. We also present results for several graph classes and constructions of graphs with given mutual-visibility and monophonic visibility numbers.

Girth Bounds for Well-VE-Dominated Graphs

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A graph is well-VE-dominated if all its inclusion-minimal vertex-edge dominating sets have the same cardinality. In this talk, we investigate how this property constrains the girth of a graph. We establish that every well-VE-dominated graph with minimum degree at least two has girth at most nine, and that this bound is sharp, as witnessed by C_9. We then explore stronger bounds under additional degree and structural assumptions, including a bound of seven for cubic graphs and sharp bounds of six and four for bipartite graphs of minimum degree at least two and three, respectively. We also show that C_9 is the unique connected subcubic well-VE-dominated graph of minimum degree at least two and girth at least nine. Finally, we discuss structural restrictions at large girth and remaining open questions.

New Convex Relaxations for Global Optimization Problems in Water Networks

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Optimization problems in water distribution networks involve nonconvex hydraulic relations and discrete decisions concerning pipe selection and pump and valve operation. In this study, we develop new convex relaxations for water network design and operation problems. We first establish that the convex hull of each hydraulic relation is power-cone representable. We then propose methods for constructing second-order cone outer approximations of power cones. For fixed binary decisions, we show that the remaining continuous problem admits at most one feasible solution and that its feasibility conditions correspond to the Karush–Kuhn–Tucker optimality conditions of a convex optimization problem. Building on these properties, we develop a cutting-plane algorithm for solving the resulting nonconvex mixed-integer nonlinear programming problems. We also introduce network-based valid inequalities and preprocessing techniques to strengthen the formulations.

Network Infection Processes and Discrete Optimization

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Invited Lecture — Saturday, 17 October 2026

Network infection processes have been studied for decades across a wide range of disciplines, including epidemiology, sociology, economics, communication networks, statistical physics, and risk management. A major milestone in this area was the introduction of the Influence Maximization problem by Kempe, Kleinberg, and Tardos in 2003, which formulated the spread of influence in social networks as a discrete optimization problem. Their work initiated a large and active research field, leading to numerous extensions and generalizations.

In this lecture, we provide an overview of the theory and applications of network infection processes from a unified perspective, with particular emphasis on the algorithmic challenges arising in discrete optimization. We will discuss both classical optimization problems defined within precise stochastic models and more realistic settings in which only partial observations of the underlying processes are available. Such weakly specified problems are common in practical applications and pose significant methodological challenges. To address these challenges, recent approaches that integrate techniques from discrete optimization, graph theory, and machine learning will be also presented.

Multimode Prize Collecting Forest Problem

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Motivated by rural electrification planning, we present the Multimode Prize Collecting Forest Problem. This problem seeks the minimum-cost forest in which each node is assigned to one of the central tree, local tree and standalone modes. Nodes assigned to the central tree mode must form a single spanning tree, while the nodes assigned to the local tree mode may form multiple spanning trees. This problem, which reduces to the Prize Collecting Steiner Tree Problem when the local tree costs are sufficiently high, is an NP-hard problem. We present flow-based and cut-based mixed integer linear programming models for this problem, and propose a branch-and-cut algorithm for the solution of the cut-based model. In addition, we propose a Modified Kruskal-based heuristic. Extensive experiments conducted on various data sets generated by drawing inspiration from real-life data sets show that the branch-and-cut algorithm considerably outperforms the flow model and the heuristic algorithm produces near optimal solutions in very short computational times. The proposed heuristic approach further improves the performance of the branch-and-cut algorithm by serving as a warm start.

A Steiner Tree Approach to Infrastructure Planning for Disaster Preparedness

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In post-disaster scenarios, connectivity between Emergency Response Centers is critical. As a Proof of Concept, this presentation proposes a Steiner Tree-based methodology to identify and evaluate the most critical links within a road network. Using the Suffolk County transportation network, the initial optimum sub-network is generated using an exact Steiner Tree solver (SCIP-Jack). This network is then analyzed by systematically removing links one by one and recalculating the new optimum routes. By proposing a new method for identifying critical roads, this approach can serve as a valuable decision-making tool for infrastructure investment and resilience planning.

Extracting Illicit-Rich Subgraph from Transaction Networks via Feed-Guided Shortest Paths

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We study investigative subgraph extraction for Anti-Money Laundering (AML) transaction graphs using the Shortest Paths Network Search Algorithm (SPNSA), which aggregates shortest paths among a small input feed of entities. Since SPNSA is highly sensitive to the feed choice, we evaluate several graph-only feed selection strategies and propose a motif-based coherent feed selection method. Experiments on the IBM-AML benchmark (HI-Small, LI-Small, HI-Medium) and the Elliptic Bitcoin transaction graph show that the proposed strategy yields markedly higher illicit ratios in the extracted subgraph than random and degree-based heuristics on Elliptic and the IBM-AML HI variants, while keeping the output compact.

Graph Distributions and Quantum Contextuality

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Graphs provide a natural combinatorial framework for describing measurement scenarios in quantum theory. By associating probability distributions to the vertices and edges of a graph, one can investigate how local probabilistic data fit together and what global constraints they satisfy. In this talk, we study graph distributions and their relation to quantum contextuality.

We begin by introducing graph distributions, which assign probability distributions to the vertices and edges of a graph subject to compatibility conditions. These constraints give rise to convex polytopes whose structure encodes the possible probabilistic models on a given measurement scenario. We then discuss how contextuality can be understood through the relationship between these distributions and deterministic models, connecting the geometric structure of the resulting polytopes to the existence of classical explanations for quantum correlations.

Using simple graph examples, we illustrate how the underlying measurement scenario influences the space of admissible distributions and how inequalities can distinguish classical from contextual behaviours. The aim is to demonstrate how combinatorial and geometric structures arising from graphs can shed light on the mathematical foundations of quantum contextuality.

Algebraic and Combinatorial Aspects of Zero-Divisor Graphs over Commutative Rings

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Let *R* be a commutative ring with identity, and let \(Z(R)\) denote its set of zero-divisors. The zero-divisor graph \(\Gamma(R)\) is the simple graph whose vertices are the nonzero elements of \(Z(R)\), with two distinct vertices adjacent precisely when their product is zero. This construction establishes a natural connection between the multiplicative structure of a ring and the combinatorial properties of an associated graph.

In this talk, we explore fundamental algebraic and combinatorial aspects of zero-divisor graphs, focusing on established relationships between ring-theoretic properties and graph invariants. Particular attention is given to connectivity, diameter, cycles, and characteristic graph configurations. We revisit classical structural results, including the connectedness of nonempty zero-divisor graphs and the upper bound of three on their diameter.

Selected examples involving finite commutative rings illustrate how algebraic multiplication influences graph structure. We also discuss classical questions concerning the realization of finite graphs as zero-divisor graphs of commutative rings.

The presentation aims to provide a rigorous perspective on the interaction between commutative algebra and structural graph theory, emphasizing established results, mathematical examples, and broader theoretical connections.

Globally Simple Heffter Arrays $H(n;k)$ with $k \equiv 1,2 \pmod 4$

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Heffter arrays are combinatorial structures used to construct orthogonal cyclic cycle decompositions and biembeddings of complete graphs onto surfaces. A Heffter array $H(m,n;h,k)$ is an $m \times n$ partially filled array with distinct nonzero entries from Z_{2nk+1} such that each row contains h filled cells, each column contains k filled cells, the elements in the filled cells form a half-set of Z_{2nk+1}, and every row and column sums to zero modulo $2nk+1$. If these row and column sums equal zero over the integers, the structure is called an integer Heffter array. Furthermore, such an array is called globally simple if the partial sums of the entries in each row and column, evaluated in their natural order, are distinct modulo $2nk+1$. When $m = n$ and $h = k$, the array is square and denoted by $H(n;k)$. While the existence of globally simple square Heffter arrays has been established for several congruence classes, the cases where $k \equiv 1,2 \pmod 4$ for $k > 10$ have remained an open problem [1]. In this work, we address this gap in the literature by explicitly constructing globally simple integer Heffter arrays $H(n;k)$ for the previously open cases. Consequently, these constructions guarantee the existence of orthogonal cyclic k-cycle decompositions of the complete graph K_{2nk+1} for these parameters. This concludes all modulo classes.

References

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Cyclic and orthogonal decompositions of K_n with a view towards stars

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An H-decomposition of K_n is a partition of its edges into copies of a graph H. In this talk, we examine H-decompositions, cyclic decompositions and orthogonal decompositions of K_n. We review classical existence results and the difference method for cyclic decompositions. Two H-decompositions are orthogonal if any two copies of H, one from each, share at most one edge. We investigate the maximum number of pairwise orthogonal H-decompositions of K_n, which, by a theorem of Caro and Yuster, tends to infinity along admissible n. Lastly, we consider the case where H is a star $K_{1,\ell}$ and demonstrate that orthogonality only restricts stars with a common centre, which leads to an upper bound on the maximum number of pairwise orthogonal star decompositions of K_n. The talk also provides background for the preceding talk on Heffter arrays.